

Constraints from precision measurements on the hadron-molecule interpretation of X, Y, Z resonances

AD Polosa^{*,¶}

^{}Dipartimento di Fisica, Sapienza Università di Roma, Piazzale Aldo Moro 2, I-00185 Roma, Italy*

[¶]INFN Sezione di Roma, Piazzale Aldo Moro 2, I-00185 Roma, Italy

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Abstract

The precise measurement of binding energy, total width and $DD\pi$ branching fraction, constrains, in a model independent way, the loosely bound hadron molecule interpretation of $X(3872)$. A full discernment is not yet possible on the basis of present data. Similar considerations might be extended to all those Y, Z resonances susceptible of a molecular description.

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The literature on the loosely bound molecule interpretation of $X(3872)$ and several other Y, Z states, is very large, see for example the references quoted in [1], and it influenced a rather wide consensus, also among experimentalists, on its validity as the correct and economical picture for the description of most of X, Y, Z states – even though it is the less appealing picture in terms of any new understanding in strong interactions dynamics.

The well known fact that the formation of prompt X s from $D\bar{D}^*$ pairs coalescing into unstable bound states is not a natural option in high energy proton-proton collisions [2], even when final state multi-body interactions are taken into account [3], should be the problem to be solved in order to explain production data, prior to any statement on loosely bound molecules phenomenology. However some authors believe that the issues on high energy production will eventually be solved resorting to strong interaction rescattering mechanisms of the sort of those proposed in [4].

On the experimental side, data on the production of (anti)deuteron at high transverse momenta – $p_T \gtrsim 8$ GeV – in LHC collisions, especially at ALICE [1], might have a strong discriminating meaning when compared to the high p_T prompt production cross section of X as measured by CMS, see *e.g.* [5]. Some simple extrapolations of available low- p_T data on anti-deuteron production at ALICE seem to suggest that we cannot expect any significant production rate at high p_T , whereas the X is copiously observed at CMS with p_T hadron cuts as large as $p_T \gtrsim 10$ GeV. If confirmed in future analyses, the lack of anti-deuterons at high p_T cuts would naturally speak against any deuteron-like interpretation of X .

In this brief report I discuss, with a standard ¹ and model-independent argument, how a precise experimental determination of the binding energy \mathcal{E} ($\mathcal{E} = m_D + m_{D^*} - m_X \gtrsim 0$), total width Γ_X and $\mathcal{B}(X \rightarrow DD\pi)$ branching fraction, would further constrain the loosely bound hadron molecule picture of $X(3872)$. Similar considerations might equally be extended to Y, Z molecular candidates. For the time being we only know that $\Gamma_X \lesssim 1.2$ MeV and $\mathcal{B}(X \rightarrow DD\pi) > 32\%$ with $m_X = 3871.69 \pm 0.17$ MeV – precise determination of D and D^* masses enter as well in the determination of the binding energy \mathcal{E} .

Consider the amplitude for the elastic process $\alpha \rightarrow \beta$. Using the $1/\sqrt{2E}$ wave function normalization for in and out particles and the appropriate $1/(2\pi)^3$ factors, one can write the scattering amplitude ² as [6]

$$f(\alpha \rightarrow \beta) = -\frac{1}{8\pi E} A_{\beta\alpha} \quad (1)$$

where E is the energy in the α state.

Suppose now that the initial and final states are two spinless particles and there is a pole dominance in the transition amplitude $A_{\beta\alpha}$, due to some intermediate state c . In the particular case of the (elastic) scattering of two hadrons a and b we have $\alpha = \beta = ab$ and

$$f(ab \rightarrow c \rightarrow ab) = -\frac{1}{8\pi E} g^2 \frac{1}{(p_a + p_b)^2 - m_c^2} \quad (2)$$

¹The quantum mechanics formulas presented in this note are standard. I review them concisely in the context of X, Y, Z phenomenology.

² $d\sigma/d\Omega = |f|^2$

where g (which has the dimensions of a mass) is proportional to the strength of the attractive force responsible for their interaction. Let c be an (unstable) bound state of a and b with binding energy $\mathcal{E} \rightarrow 0^+$. We have

$$m_c \simeq m_a + m_b - \mathcal{E} \quad (3)$$

and, in the non-relativistic approximation of slowly recoiling a, b

$$E \simeq m_a + m_b \quad (4)$$

whereas we expand

$$(p_a + p_b)^2 \simeq (m_a + m_b + T)^2 \quad (5)$$

where $T = \mathbf{p}_a^2/2m_a + \mathbf{p}_b^2/2m_b = \mathbf{p}^2/2m$ is the, supposedly small, recoil kinetic energy in the center-of-mass of the ab pair, where $\mathbf{p}_a = -\mathbf{p}_b$, and m is the ab reduced mass. Plugging (3), (4) and (5) in (2) we have

$$f(ab \rightarrow c \rightarrow ab) \simeq -\frac{1}{16\pi(m_a + m_b)^2} g^2 \frac{1}{\mathcal{E} + T} \quad (6)$$

neglecting small \mathcal{E}^2 and T^2 terms.

In the non-relativistic quantum mechanics description of resonant low energy scattering, see [7], two slow particles ($kR \ll 1$) interacting through an attractive potential U (of range R), with a superficial discrete level at $-\mathcal{E}$ ($|\mathcal{E}| \ll |U|$), have a universal elastic scattering amplitude

$$f(ab \rightarrow ab) = -\frac{1}{\sqrt{2m}} \frac{\sqrt{\mathcal{E}} - i\sqrt{T}}{\mathcal{E} + T} \quad (7)$$

which is independent of the details of the potential U , being affected only by the value of the resonant level at $-\mathcal{E}$. A brief proof of this formula is provided below.

The kinetic energy T in the latter formula, coincides with the total energy $T = E > 0$ in region II ($r > R$) outside the potential range, where free motion takes place with wave function $\chi(r)_{II} \sim \sin(kr + \delta_0)$ – given that we assume $kR \ll 1$, $\chi(r)_{II}$ varies slowly as $r \rightarrow 0$. Because of the slow variation of χ_{II} , the matching condition $(\chi'/\chi)_{II} = (\chi'/\chi)_I$, to be taken at some $r^* > 0$ point, could formally be computed at $r^* = 0$. Therefore we obtain $(\chi'/\chi)_{II} = k \cot \delta_0$.

Within region I ($r < R$), the Schrödinger equation will not depend explicitly on energy, as $U \gg |\mathcal{E}|$, and the boundary condition will not depend on the total energy either. Given the independency on E in region I , let us choose to be at the stationary state. In correspondence of a generic boundary value r^* , we have that $\chi_I = A \sin k_I r^* \equiv B e^{-\kappa r^*}$ and $\kappa = \sqrt{2m\mathcal{E}}/\hbar$. We choose the latter form for the boundary condition so to be independent on U and r^* : $(\chi'/\chi)_I = -\kappa$.

Since k in region II is $k = \sqrt{2mT}/\hbar$, the boundary condition at r^* is $\cot \delta_0 = -\sqrt{\mathcal{E}/T}$. The latter formula can be used in the S-wave scattering amplitude ³ leading eventually to Eq. (7).

³

$$f(\alpha \rightarrow \beta) = \frac{1}{k(\cot \delta_0 - i)} \quad (8)$$

From these considerations, the determination of the scattering length a follows⁴ and a comparison between Eq. (7) and (6), the latter encoding the dependency on the interaction force in the g coupling, leads to the following relation

$$\mathcal{E} \simeq \frac{g^4}{512\pi^2} \frac{m^5}{(m_a m_b)^4} \quad (9)$$

which, again, is independent on the details of U .

In the case of the $X(3872)$ loosely bound molecule, we should take $m_a = m_D$, $m_b = m_{D^*}$ and define the strong coupling g through⁵

$$\langle D^0 \bar{D}^{0*}(\epsilon, q) | X(\lambda, P) \rangle = g \lambda \cdot \epsilon^* \quad (10)$$

In principle g^2 is derived from the $\Gamma(X \rightarrow D\bar{D}^*)$ decay width, which is $\Gamma \sim g^2 \Phi$ if particles are considered to be spinless (Φ being the two-body decay phase space). Taking into account the spin of D^* and X , one should rather substitute in (9)

$$g^2 \rightarrow g^2 \frac{1}{3} \left(2 + \frac{(m_X^2 + m_{D^*}^2 - m_D^2)^2}{4m_X^2 m_{D^*}^2} \right) \quad (11)$$

which, however, turns out to be numerically $\simeq g^2$. The actual value of g is extracted from data on the branching ratio $\mathcal{B}(X \rightarrow DD\pi)$, which is measured experimentally to be larger than 32% [9]. However the total width is poorly known, as $\Gamma_X \lesssim 1.2$ MeV. Using these two extreme values and the $X \rightarrow DD\pi$ decay rate

$$\begin{aligned} \Gamma(X \rightarrow DD\pi) &= \frac{1}{3} \frac{1}{8\pi m_X^2} 3(g\sqrt{2})^2 p^*(m_X^2, m_D^2, s) \times \\ &\times \frac{1}{\pi} \frac{s/m_{D^*} \Gamma_{D^*} \mathcal{B}(D^* \rightarrow D\pi)}{(s - m_{D^*}^2)^2 + (s/m_{D^*} \Gamma_{D^*})^2} \frac{m_{D^*}}{\sqrt{s}} \frac{p^*(s, m_D^2, m_\pi^2)}{p^*(m_{D^*}^2, m_D^2, m_\pi^2)} \end{aligned} \quad (12)$$

where the decay momentum is $p^*(x, y, z) = \sqrt{\lambda(x, y, z)}/2\sqrt{x}$, λ being the Källén triangular function, it is found that $g \approx 4$ GeV [10].

Considering for example a branching fraction of $\mathcal{B}(X \rightarrow DD\pi) \simeq 0.32$, we obtain $\mathcal{E} = \mathcal{E}_{\text{exp}}$ on assuming a total width of the X as large as ≈ 300 KeV: lower values of Γ_X would also be possible for higher branching ratios $\mathcal{B}(X \rightarrow DD\pi)$, whereas higher Γ_X values are excluded by (9): see shaded areas in Fig. 1.

In all cases we see that only very small kinetic energies are allowed in the $D\bar{D}^*$ system, definitely smaller than the conservative upper limit on the relative momentum in the center of mass of the

⁴ At very low energies, $k \sim 0$, in region II we actually have to solve $\chi''_{II} = 0$, which has the solution $\chi_{II} \sim (r - a)$ (infinitely long-wave-limit of a sin function). Therefore it also holds that $\lim_{k \rightarrow 0} k \cot \delta_0 = \lim_{k \rightarrow 0} (\chi'/\chi)_{II} = -1/a$, which defines the scattering length a . From the matching condition found above, $1/a = \kappa$ so that $a = \hbar/\sqrt{2m\mathcal{E}}$ – as it is also found discussing the Low equation as described in [8] – and $\sigma = 4\pi a^2 = 2\pi\hbar^2/m\mathcal{E}$.

⁵ Since X has positive charge conjugation, the final state is $|f\rangle = (|D^0 \bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}$. When extracting g defined in (10) from data a factor of $\sqrt{2}$ has to be included: $g \rightarrow \sqrt{2}g$.

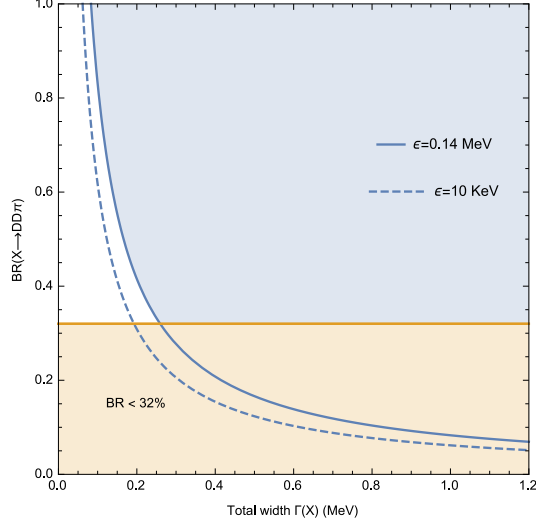


Figure 1: Given the experimentally excluded region (shaded), a loosely bound molecule compatible with (9), has total width Γ_X , branching ratio $\mathcal{B}(X \rightarrow DD\pi)$ and binding energy $\mathcal{E} = m_D + m_{D^*} - m_X$ as in the plot – here $T \approx -\mathcal{E} \approx 0$. This plot is obtained using the approximate values $m_X = 3871.66$ MeV, $m_D = 1864.84$ MeV and $m_{D^*} = 2006.96$ MeV for $\mathcal{E} = 0.14$ MeV and a slightly modified value for $m_X = 3871.79$ MeV in order to get $\mathcal{E} = 10$ KeV.

pair ($k_{\text{rel}} \lesssim 50$ MeV) which was used in [2]. This further suggests that hadronization of prompt X at LHC cannot proceed through a DD^* coalescing phenomenon, the Monte Carlo estimate of the slowly recoiling DD^* being very adverse to the measured high cross section.

In a deuteron-like description of $X(3872)$ based on a (three-dimensional) square well potential of depth $U_0 \simeq 9$ MeV and range $R \simeq 3$ fm, a bound state energy $B = |\mathcal{E}_{\text{exp}}| = 0.1$ MeV is obtained. The expectation value of the kinetic energy in the bound state ψ is found to be

$$\langle T \rangle_\psi \simeq 4 \text{ MeV} \quad k_{\text{rel}} \simeq 86 \text{ MeV} \quad (13)$$

a rather higher value of T with respect to those discussed before. However we have to observe that, in this model, the D and \bar{D}^* mesons have indeed *finite* negative total energy. To make this happen, the DD^* pair produced in pp hadronization must interact with at least a third hadron to change its relative kinetic energy and fall in the discrete (even though superficial) level of the attractive potential. The expected X width would therefore be $\Gamma_X \approx \Gamma_{D^*} \approx 100$ KeV, even at a binding energy as large as $\mathcal{E} = 0.14$ MeV (compare to Fig 1), for it would be a stable bound state whose lifetime coincides with the lifetime of the shortest lived between its components. Monte Carlo studies on πDD^* final state rescatterings do not encourage this picture either [3], suggesting overall that hadronization should most likely produce compact tetraquarks, which might otherwise be understood in the picture described by [11].

The formation of X as a loosely DD^* bound state might occur either via a low energy ($T \approx 0$) resonant scattering mechanism (see Fig. 1) or via multi-body final state interactions producing a deuteron-like state. The latter case, although not supported by Monte Carlo simulations [3], might

be more realistic when considering X prompt production in high energy pp collisions at the LHC, with high transverse momentum cuts on hadrons.

A number of other states are described in the literature as loosely bound hadron molecules: notably the charged $Z_b(10610)$ and $Z'_b(10650)$ [12] happen to be very close to $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, whereas their analogs in the charm sector, $Z_c(3900)$ and $Z'_c(4025)$ [13], are less compelling molecules for their binding energy turns out to be $-\mathcal{E} > 0$ by about 20 MeV. Accessing precise measurements of their properties will enable to further test the relation between binding energy and partial widths as done in Fig. 1 for the X . In consideration of the multitude of thresholds that can be formed combining all known open charm and beauty mesons, there are also a number of Y resonances which are eligible molecular candidates.

Although the interpretation of the X in terms of a loosely bound molecule is just given for granted by many, we have to remark that it is challenged by diverse constraints as those discussed in the literature on high energy production [2–4] or those implicit in Eq. (9), and sketched in Fig. 1. There will hopefully be a number of more precise experimental measurements on the properties of X, Y, Z resonances which will help in disentangling this intricate matter.

Note Added. Soon after the preparation of this draft, I noticed the paper by Tomaradze *et al.* [14] claiming that a precision measurement of the mass difference between D^0 and D^{*0} mesons leads to a binding energy of $\mathcal{E} \sim 3 \pm 192$ KeV in the $D\bar{D}^*$ molecule interpretation of $X(3872)$.

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